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Cyclotron-transition linewidths in Ge with anisotropic scattering

Yong Jai Cho†, Nam Lyong Kang†, Kyu Soon Bae†, Jai Yon Ryu‡ and Sang Don Choi†

† Department of Physics, Kyungpook National University, Taegu 702-701, Republic of Korea§
‡ Department of Physics, Cheju National University, Cheju 690-756, Republic of Korea

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Abstract. Utilizing the state-independent projection technique, we obtain a general formula for the cyclotron-transition linewidth for anisotropic deformation potential phonon scattering. The formula is applied to calculate the width in Ge at low temperatures. It is shown that the transition between the two lowest adjacent Landau levels makes the most contribution to the temperature and the magnetic field dependence of the width, and the result agrees better with the experiment as the higher-order contributions are included. The width increases with the magnetic field, implying that the interaction of electrons with acoustic phonons increases with the field.

1. Introduction

Transport phenomena for electrons and holes in non-polar semiconductors are determined by interactions with acoustic vibrations of the lattice as well as by the scattering of impurities or other lattice defects. In particular, the interaction of the carriers with the acoustic mode of the lattice through deformation potentials generally predominates in the relatively pure materials in which the concentration of carriers is small. Bardeen and Shockley (1950) showed that for the non-degenerate and spherical energy surfaces in the Brillouin zone the lattice scattering is determined by the shifts in the energy bands resulting from dilation associated with longitudinal acoustic waves. Herring and Vogt (1956), generalized this theory to include anisotropic scatterings of the acoustic phonons for many-valley semiconductors.

The conduction bands of Ge and Si have many equivalent energy surfaces in the Brillouin zone. Because the constant-energy surfaces are ellipsoidal, shear strains as well as dilation strains can produce the deformation potentials. This shear strain leads to a term that depends on the direction of the phonon wavevector, and band-edge shifts are expected to depend on all six components of the shear tensor. Thus there might be as many as six deformation potentials. However, the ellipsoidal energy surfaces of Ge and Si are centred on the high-symmetry $\langle 111 \rangle$ and $\langle 100 \rangle$ axes, respectively, so the symmetry properties allow a reduction to just two independent potentials (Herring and Vogt 1956). These are the dilation potential Ξ_d and the uniaxial shear potential Ξ_u.

The deformation potentials for Ge and Si can be determined by utilizing measurements of the piezoresistance and magnetoresistance (Morin *et al* 1957, Herring *et al* 1959, Aubrey *et al* 1963, Fritzsche 1959, 1960), acoustoelectric effect (Weinreich *et al* 1959),

§ Fax: 82-053-952-1739; e-mail: choisd@knuhep.kyungpook.ac.kr.

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influence of uniaxial stress on the indirect absorption edge (Balslev 1966, 1967), free-carrier piezobirefringence (Riskaer 1966), cyclotron resonance (CR) (Bagguley *et al* 1962, Ito *et al* 1964, Stradling and Zhukov 1966, Murase *et al* 1970), optical determination of donor impurities (Reuszer and Fisher 1964, Wilson 1964), electronic effect on the elastic constant (Hall 1965), etc. The deformation potentials obtained by the above methods have different values in the range given by $\Xi_u = 16.0$ to 20 eV and $\Xi_d = -10.2$ to -13.5 eV for Ge, while $\Xi_u = 7.7$ to 11.3 eV and $\Xi_d = -3.4$ -6.0 eV for Si. The values for each of the cases are different beyond their experimental errors.

In the CR experiments, the cyclotron-resonance linewidth (CRLW) was measured when the magnetic field was applied parallel or perpendicular to the major ellipsoidal axis of a valley. We shall denote the half-CRLWs of the CR absorption spectrum as $\xi_\parallel$ for the parallel direction and $\xi_\perp$ for the perpendicular one. It is general that the CRLWs are written in terms of the corresponding collision times—that is, $\xi_\parallel \equiv 1/\tau_1$ and $\xi_\perp \equiv 1/\tau_2$. For many-valley semiconductors with anisotropic scattering, Herring and Vogt (1956) evaluated the relaxation times which reduced to $\tau_\parallel$ and $\tau_\perp$. In subsequent papers (Bagguley *et al* 1962, Ito *et al* 1964, Stradling and Zhukov 1966, Murase *et al* 1970) on the CRLW, to get the theoretical expressions for the relaxation times for anisotropic phonon scattering, the relations between the collision times for the half-CRLWs and the Herring–Vogt relaxation times given by (Bagguley *et al* 1962)

$$\frac{1}{\tau_1} = \frac{1}{\tau_\perp} \tag{1.1}$$

and

$$\frac{1}{\tau_2} = \frac{1}{2}\left(\frac{1}{\tau_\perp} + \frac{1}{\tau_\parallel}\right) \tag{1.2}$$

were adopted. The anisotropic ratio $\tau_\parallel/\tau_\perp$ is represented in terms of just the ratio ($D \equiv \Xi_d/\Xi_u$) of the deformation potential constants (Herring and Vogt 1956). Therefore, the ratio D of the deformation potential constants could be determined from the anisotropy ratio for the measured half-CRLWs through the relations (1.1) and (1.2). Using this value of D, the other deformation potential constant could be determined from fitting a calculated relaxation time to the corresponding experimental data. However, these methods for getting the two deformation potential constants encountered difficulty due to the overlapping between the absorption spectrum of the heavy hole and that of the electron, because the experiments were performed for the so-called classical limit ($k_B T \geqslant \hbar\omega$, where T is the temperature and ω the frequency of the applied eletromagnetic wave). Another problem is that the measured CRLW, in addition to a temperature dependence, also has a slight dependence on frequency for both Si (Hensel 1963) and Ge (Kobori *et al* 1990). Thus the ratio D can only be obtained by using the Herring–Vogt equations for the relaxation times, which have no frequency dependence.

In a recent paper (Cho and Choi 1996), we derived a theory of CRLWs based on the state-independent projection technique and applied the theory to obtain CRLWs of Ge for the lowest-level transition in the quantum limit ($k_B T \ll \hbar\omega$). The theoretical results were obtained for the acoustic deformation potential scattering and were compared with the corresponding experimental data for Ge. From fitting the theoretical results to the experimental data, we determined the uniaxial and dilation deformation potentials for the anisotropic band model as $\Xi_u = 18.0 \pm 0.6$ eV, $\Xi_d = -12.2 \pm 0.68$ eV. However, the line-shape formula used in our work (Cho and Choi 1996) was formulated for the temperature region of the quantum limit and thus application to real systems is limited.

In the present work, we will generalize the theory of Cho and Choi (1996) to include the cyclotron transitions between higher Landau levels for anisotropic scattering. The present theory is based on the state-independent projection technique, and thus is quite different to our previous ones (Choi and Chung 1983, Cho and Choi 1993, 1994). The linewidths can be obtained straightforwardly without direct connection with the power function, and the anisotropy of the material structure can be dealt with easily (see section 3). In this sense, this theory is more or less similar to the theory of Peeters and Devreese (Peeters and Devreese 1983, Wu *et al* 1986). In section 2, we will reformulate the theory in such a way that all of the higher-order contributions can be taken into account. In section 3, we will apply the theory to the anisotropic electron–phonon scattering, and in section 4, we will calculate the half-CRLWs of Ge. The temperature and magnetic field dependences will be investigated in comparison with the existing experimental data. Except for the inclusion of higher-order transitions, the approach is similar to that adopted in our previous paper (Cho and Choi 1996). Section 5 will be devoted to concluding remarks.

2. The cyclotron resonance line-shape formula

In this section we will generalize the lineshape formula based on the state-independent projection technique already introduced in previous work (Cho and Choi 1996).

At low temperatures, the transport phenomena of electrons in non-polar semiconductors such as pure Ge and pure Si are generally determined by the scattering of acoustic phonons. We consider such a system as an electron–phonon interacting system whose Hamiltonian is given by

$$H_{\mathrm{eq}} = H_{\mathrm{el}} + H_{\mathrm{ph}} + H_{\text{el-ph}}. \tag{2.1}$$

Each part of this Hamiltonian is expressed, in the second-quantized formalism, as

$$H_{\mathrm{el}} = \sum_{\alpha} \langle \alpha | h_{\mathrm{el}} | \alpha \rangle a_{\alpha}^{+} a_{\alpha} \tag{2.2}$$

$$H_{\mathrm{ph}} = \sum_{\boldsymbol{q},s} \hbar \omega_{\boldsymbol{q},s} b_{\boldsymbol{q},s}^{+} b_{\boldsymbol{q},s} \tag{2.3}$$

$$H_{\text{el-ph}} = \sum_{\boldsymbol{q},s} \sum_{\alpha,\mu} V_{\boldsymbol{q},s} \langle \alpha | \exp(\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{r}) | \mu \rangle a_{\alpha}^{+} a_{\mu} (b_{\boldsymbol{q},s} + b_{-\boldsymbol{q},s}^{+}) \tag{2.4}$$

where a_{α}^{+} (a_{α}) and $b_{\pm\boldsymbol{q},s}^{+}$ ($b_{\pm\boldsymbol{q},s}$), respectively, denote the creation (annihilation) operators for an electron in the state $|\alpha\rangle$ and for a phonon in the state $|\pm\boldsymbol{q}, s\rangle$, $\hbar\omega_{\boldsymbol{q},s}$ is the phonon energy, $\boldsymbol{q}$ the phonon wave vector, s the index of the phonon mode, and $V_{\boldsymbol{q},s}$ the coupling coefficient for the electron–phonon interaction.

The Ge and Si conduction bands have many equivalent ellipsoidal energy surfaces in the first Brillouin zone. If a static magnetic field $\boldsymbol{B}$ is applied along the major axis of an ellipsoid (the z-direction), the eigenvalue of the single-electron Hamiltonian h_{el} in equation (2.2) is given by

$$h_{\mathrm{el}} |\alpha\rangle = (\mathcal{E}_{N_\alpha, k_{z\alpha}} + \mathcal{E}_C) |\alpha\rangle. \tag{2.5}$$

Here $|\alpha\rangle \equiv |N_\alpha, k_{y\alpha}, k_{z\alpha}\rangle$ is the Landau state of the level index N_α and the electron wave vector $\boldsymbol{k}_\alpha$. Also $\mathcal{E}_C$ is the bottom of the conduction band, and $\mathcal{E}_{N_\alpha, k_{z\alpha}}$ the Landau energy level given by

$$\mathcal{E}_{N_\alpha, k_{z\alpha}} = (N_\alpha + 1/2)\hbar\omega_c + \hbar^2 k_{z\alpha}^2 / 2m_l \tag{2.6}$$

where ω_c ($\equiv eB/m_t$) is the CR frequency of the electrons and m_l (m_t) the longitudinal (transverse) effective mass of the electron.

When a circularly polarized electromagnetic wave with frequency ω and electric field amplitude E_0 is incident upon the electron–phonon system along the z-axis, the absorption power density of the electrons is given by

$$\mathcal{P}(\omega, \omega_c, T) \;=\; \mathcal{P}_{\max}(\omega, T)\frac{\xi(\omega, T)^2}{(\omega-\omega_c)^2+\xi(\omega, T)^2}. \tag{2.7}$$

Here the maximum absorption power intensity is

$$\mathcal{P}_{\max}(\omega, T) = \frac{e^2E_0^2n(\omega, T)}{m_t\xi(\omega, T)} \tag{2.8}$$

where the number density of electrons which participate in constructing the absorption peak is

$$n(\omega, T) = \frac{g_v\omega(2\pi m_l m_t^2 k_{\rm B}T)^{1/2}\exp[-(\mathcal{E}_C-\zeta)/k_{\rm B}T]}{4\pi^2\overline{h}^2\sinh(\overline{h}\omega/2k_{\rm B}T)}. \tag{2.9}$$

Here, ζ is the chemical potential for an electron, and the averaged relaxation rate is

$$\begin{aligned}\xi(\omega, T) = {} & \frac{\overline{h}}{(2\pi m_l k_{\rm B}T)^{1/2}}[1-\exp(-\overline{h}\omega/k_{\rm B}T)]^2\sum_{N=0}^{\infty}(N+1)\exp(-N\overline{h}\omega/k_{\rm B}T) \\ & \times \int_{-\infty}^{\infty}{\rm d}k_z\,\exp(-\overline{h}^2k_z^2/2m_lk_{\rm B}T)\gamma(\omega, T, N, k_z)\end{aligned} \tag{2.10}$$

in which the energy-dependent relaxation rate is

$$\begin{aligned}\gamma(\omega, T, N, k_z) = {} & \frac{g_v}{2\pi\overline{h}}\sum_s\sum_{N'=N,N+1}\int_0^{\infty}{\rm d}q_\perp\int_{-\infty}^{\infty}{\rm d}q_z\;q_\perp|V_{\boldsymbol{q},s}|^2K(N, N'; t) \\ & \times\Big\{N_{\boldsymbol{q},s}\delta[(1-N'+N)\overline{h}\omega-\overline{h}^2(q_z^2+2k_zq_z)/2m_l+\overline{h}\omega_{\boldsymbol{q},s}] \\ & +(N_{\boldsymbol{q},s}+1)\delta[(1-N'+N)\overline{h}\omega-\overline{h}^2(q_z^2-2k_zq_z)/2m_l-\overline{h}\omega_{\boldsymbol{q},s}]\Big\}.\end{aligned} \tag{2.11}$$

(For a derivation, see equations (5)–(11) of Cho and Choi (1996).) Here g_v is the degeneracy due to the equivalent valleys, $q_\perp \equiv (q_x^2+q_y^2)^{1/2}$, $t \equiv \overline{h}q_\perp^2/2m_t\omega$, $K(N, N'; t)$ is the K-matrix (Cho and Choi 1994), and $N_{\boldsymbol{q},s}$ is the Bose–Einstein distribution function for phonons with energy $\overline{h}\omega_{\boldsymbol{q},s}$. In equation (2.11), we have excluded the part with $|N'-N|>1$ since its contribution is negligible in conparison with the part with $|N'-N|=0$ and 1. The conduction electron states of pure semiconductors are non-degenerate and the Fermi energy is nearly in the middle of the bottom of the conduction band and the top of the valence band. Thus in equation (2.10), the electron distribution for the Landau levels, constructed above the bottom of the conduction band, is proportional to the factor of $\exp[-N\overline{h}\omega_c/k_BT]$, where N is the Landau level index. Therefore, the cyclotron resonance is mainly controlled by the electron transitions between the two lowest Landau levels. However, for metals and doped semiconductors with degenerate electron states, the cyclotron resonance is mainly controlled by the electron transitions between two neighbouring Landau levels near the Fermi surface.

Considering the energy and momentum conservation from the δ-functions, we see that the first and second terms of equation (2.11) represent the absorption and emission processes of phonons, respectively, when the electrons in Landau level N absorb the photon energy $\overline{h}\omega$ and make a cyclotron transition to the excited states of Landau level N'. In equations (2.9) and (2.11), g_v is inserted since each CR absorption power peak is constructed only by the electrons of the equivalent valleys with the same inner angle between the major axes of the valleys and the direction of the magnetic field $\boldsymbol{B}$. When the magnetic field is applied along the major axes of the valleys—that is, $\langle 111\rangle$ for Ge—two peaks appear. If we are

concerned only about the absorption peak for the cyclotron effective mass of m_t, $g_v = 1$ for Ge. Note that the averaged relaxation rate of equation (2.10) is equal to the half-CRLW, since the spectrum of the absorption power intensity for ω_c has a form of Lorentzian shape.

3. The averaged relaxation rate for anisotropic electron–phonon scattering

In anisotropic solids such as Ge and Si, the velocities of acoustic waves are different for different directions of propagation. However, the difference is not especially large (Bardeen and Shockley 1950), and thus we assume that the speed of sound is independent of the direction of propagation. Then we can define the coupling coefficient for the longitudinal mode as

$$V_{\boldsymbol{q},L} = \mathrm{i}(\hbar q/2\rho_m \bar{v}_L)^{1/2}(\Xi_d + \Xi_u \cos^2\theta_{\boldsymbol{q}}) \tag{3.1}$$

and that for the sum of two branches of the transverse mode as

$$V_{\boldsymbol{q},T} = \mathrm{i}(\hbar q/2\rho_m \bar{v}_T)^{1/2}\Xi_u \sin\theta_{\boldsymbol{q}} \cos\theta_{\boldsymbol{q}} \tag{3.2}$$

where $\theta_{\boldsymbol{q}}$ is the inner angle between the major ellipsoidal axis and the phonon wave vector $\boldsymbol{q}$, and ρ_m is the mass density of the bulk. Also, the average speed of sound $\bar{v}_L$ ($\bar{v}_T$), or the average speed of the longitudinal (transverse) acoustic phonon, can be defined as $\bar{v}_L = (c_L/\rho_m)^{1/2}$ and $\bar{v}_T = (c_T/\rho_m)^{1/2}$ in which the average elastic constants are given by $c_L = (3c_{11} + 2c_{12} + 4c_{44})/5$ and $c_T = (c_{11} - c_{12} + 3c_{44})/5$ where the c_{ij} are the elastic stiffness constants (Ridley 1988). Ξ_u and Ξ_d are the uniaxial and the dilation potential constants. In this scheme the state-dependent relaxation rate of equation (2.11) is rewritten as

$$\gamma(\omega, T, N, k_z) = \gamma_L^{(+)} + \gamma_L^{(-)} + \gamma_T^{(+)} + \gamma_T^{(-)} \tag{3.3}$$

where the subscript L (T) and the superscript $+$ ($-$), respectively, represent the longitudinal (transverse) mode of phonons and the phonon absorption (emission) process. If the direction (the z-axis) of the magnetic field is arranged along the major axis of a valley, $\sin\theta_{\boldsymbol{q}}$ and $\cos\theta_{\boldsymbol{q}}$ can be replaced by $q_\perp/q$ and q_z/q, respectively. Therefore, by taking into account equations (3.1) and (3.2) in equation (2.11), each part of equation (3.3) is given by

$$\gamma_L^{(\pm)} = \frac{g_v \Xi_u^2}{4\pi\rho_m \bar{v}_L} \sum_{N'=N,N+1} \int_0^\infty \mathrm{d}q_\perp \int_{-\infty}^\infty \mathrm{d}q_z \; q q_\perp [D + (q_z/q)^2]^2 K(N, N'; t) \times (N_{q,L} + 1/2 \mp 1/2)\delta[(1 - N' + N)\hbar\omega - \hbar^2(q_z^2 \pm 2k_z q_z)/2m_l \pm \hbar\bar{v}_L q] \tag{3.4}$$

and

$$\gamma_T^{(\pm)} = \frac{g_v \Xi_u^2}{4\pi\rho_m \bar{v}_T} \sum_{N'=N,N+1} \int_0^\infty \mathrm{d}q_\perp \int_{-\infty}^\infty \mathrm{d}q_z \; (q_\perp^3 q_z^2/q^3) K(N, N'; t) \times (N_{q,T} + 1/2 \mp 1/2)\delta[(1 - N' + N)\hbar\omega - \hbar^2(q_z^2 \pm 2k_z q_z)/2m_l \pm \hbar\bar{v}_T q] \tag{3.5}$$

where $D \equiv \Xi_d/\Xi_u$. To obtain the averaged relaxation rate, we should perform the triple integrations in equations (3.4) and (3.5). The $q_\perp$-integration is trivial, since its integrand has a δ-function, but the other integrations are very complex. Therefore, we shall first obtain the analytical solution for the $q_\perp$-part, and then the numerical calculations of the rest through computer work.

To carry out the integrations over $q_\perp$ in equations (3.4) and (3.5), we first obtain the roots of the arguments of the δ-functions as

$$q_L^{\pm} = \left\{ \left[(1 - N' + N)\bar{h}\omega - \bar{h}^2(q_z^2 \pm 2k_z q_z)/2m_l \right]^2 / (\bar{h}\bar{v}_L)^2 - q_z^2 \right\}^{1/2} \tag{3.6}$$

and

$$q_T^{\pm} = \left\{ \left[(1 - N' + N)\bar{h}\omega - \bar{h}^2(q_z^2 \pm 2k_z q_z)/2m_l \right]^2 / (\bar{h}\bar{v}_T)^2 - q_z^2 \right\}^{1/2} \tag{3.7}$$

respectively. These solutions have to be valid within the ranges

$$\mp(1 - N' + N)\bar{h}\omega \pm \bar{h}^2(q_z^2 \pm 2k_z q_z)/2m_l \geqslant 0 \tag{3.8}$$

determined from the arguments of the δ-functions in equations (3.4) and (3.5) and

$$[(1 - N' + N)\bar{h}\omega - \bar{h}^2(q_z^2 \pm 2k_z q_z)/2m_l]^2 / (\bar{h}\bar{v}_L)^2 - q_z^2 \geqslant 0 \tag{3.9}$$

$$[(1 - N' + N)\bar{h}\omega - \bar{h}^2(q_z^2 \pm 2k_z q_z)/2m_l]^2 / (\bar{h}\bar{v}_T)^2 - q_z^2 \geqslant 0 \tag{3.10}$$

determined from the arguments of the square roots in equations (3.6) and (3.7), respectively.

Then performing the integrations over $q_\perp$ in equations (3.4) and (3.5), the results are given by

$$\gamma_L^{(\pm)} = \frac{g_v \Xi_u^2}{4\pi\rho_m \bar{h}\bar{v}_L^2} \sum_{N'=N,N+1} \int_{a_1^{\pm}}^{a_2^{\pm}} \mathrm{d}q_z \; (q_L^{\pm 2} + q_z^2) \left(D + \frac{q_z^2}{q_L^{\pm 2} + q_z^2} \right)^2 \times K(N, N'; \bar{h}q_L^{\pm 2}/2m_t\omega)[N(q_L^{\pm}, q_z) + 1/2 \mp 1/2] \tag{3.11}$$

and

$$\gamma_T^{(\pm)} = \frac{g_v \Xi_u^2}{4\pi\rho_m \bar{h}\bar{v}_T^2} \sum_{N'=N,N+1} \int_{b_1^{\pm}}^{b_2^{\pm}} \mathrm{d}q_z' \frac{q_T^{\pm 2} q_z'^2}{q_T^{\pm 2} + q_z'^2} K(N, N'; \bar{h}q_T^{\pm 2}/2m_t\omega) \times [N(q_T^{\pm}, q_z) + 1/2 \mp 1/2] \tag{3.12}$$

where

$$N(q_{L(T)}^{\pm}, q_z) = 1/[\exp\{\bar{h}\bar{v}_{L(T)}(q_{L(T)}^{\pm 2} + q_z^2)^{1/2}/k_B T\} - 1].$$

Also, the a_i^+ (a_i^-) are determined by the upper (lower) parts of equations (3.8) and (3.9), and b_i^+ (b_i^-) by the upper (lower) parts of equations (3.8) and (3.10).

4. The half-cyclotron-resonance linewidth of Ge

The Ge conduction band has four ellipsoidal energy surfaces along the $\langle 111 \rangle$ axes at the L point in the first Brillouin zone. Among the values of the physical parameters for Ge, the elastic stiffness constants (Ferry 1991) are given by $c_{11} = 1.29 \times 10^{11}$ N m^{-2}, $c_{12} = 0.48 \times 10^{11}$ N m^{-2}, and $c_{44} = 0.67 \times 10^{11}$ N m^{-2}. The other values are given in the paper of Cho and Choi (1994).

In order to obtain the temperature and magnetic field dependence of the half-cyclotron-resonance linewidth, we need to determine the two deformation potentials. In our recent paper (Cho and Choi 1996), we showed that the two deformation potentials could be determined by fitting the CRLW calulated in the quantum limit to the corresponding experimental data. In the quantum limit, most of the conduction electrons reside on the lowest Landau level. So we shall take into account only the transitions from the lowest Landau states $|0, k_z\rangle$ to the excited states $|0, k_z \pm q_z\rangle$ or $|1, k_z \pm q_z\rangle$. Then N in

equations (3.11) and (3.12) is set as $N = 0$. In this case, the averaged relaxation rate of equation (2.10) in the quantum limit ($k_B T \ll \hbar\omega$) can be approximated as

$$\xi(\omega, T)_{\mathrm{QL}} \simeq \frac{\hbar}{(2\pi m_l k_{\mathrm{B}} T)^{1/2}} \int_{-\infty}^{\infty} \mathrm{d}k_z \, \exp(-\hbar^2 k_z^2 / 2 m_l k_{\mathrm{B}} T) \gamma(\omega, T, 0, k_z) \tag{4.1}$$

of which the form is simply a Boltzmann average of the state-dependent relaxation rate $\gamma(\omega, T, 0, k_z)$. Using equation (4.1), we determine ratio of the two deformation potentials as $D = -0.68 \pm 0.03$ from fitting the anisotropy ratio for CRLWs to the experimental data (Murase *et al* 1970) and the uniaxial deformation potential as $\Xi_u = 18.0 \pm 0.6$ eV from fitting the temperature dependence of the half-CRLW to the experimental data (Kobori *et al* 1990) for λ (wavelength) $= 220$ μm.

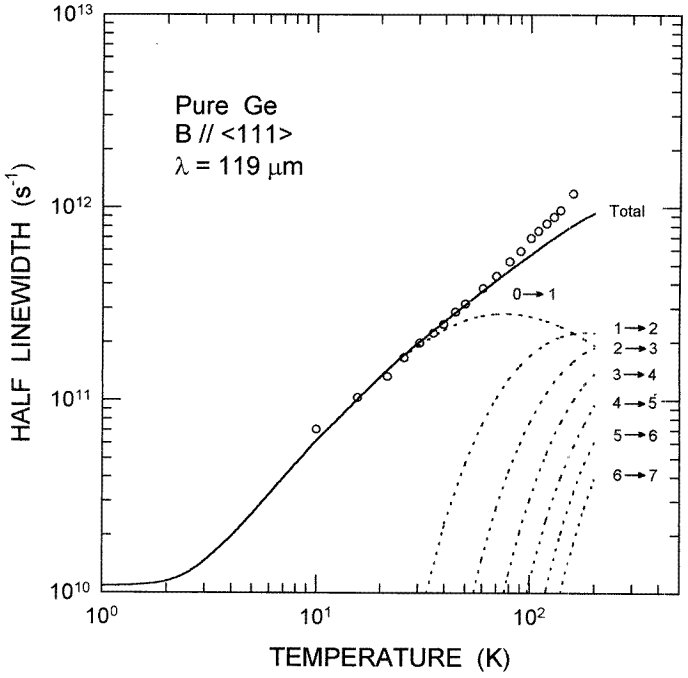

Figure 1. The temperature dependence of the half-CRLW of pure Ge at $\lambda = 119$ μm and with the magnetic field along the $\langle 111 \rangle$ direction. The solid line shows the total half-CRLW. The dotted lines denote the half-CRLWs due to the cyclotron transitions between the Landau levels N and $N + 1$. The open circles show the experimental data of Kobori *et al* (1990).

The temperature dependence of the half-linewidth is shown in figure 1. The symbol $0 \rightarrow 1$ represents the half-linewidth for the cyclotron transition in the quantum limit and the others ($N \rightarrow N + 1$, for $N \geqslant 1$) represent those for the cyclotron transitions between the higher Landau levels. With the value of D obtained above we can fit the solid line of the total half-CRLW to the experimental data (Kobori *et al* 1990) for $\lambda = 119$ μm and then obtain the uniaxial deformation potential as $\Xi_u = 16.7 \pm 0.51$ eV. For the temperatures below about 50 K, the half-CRLW formula for the quantum limit describes the experimental data well. However, for the classical-limit region, the difference between the quantum-limit theoretical values and the experimental data increases. When the cyclotron transitions for the higher Landau levels are considered, the theoretical result is improved but there still

exists some discrepancy, which may stem from the scattering of the longitudinal optical deformation potential.

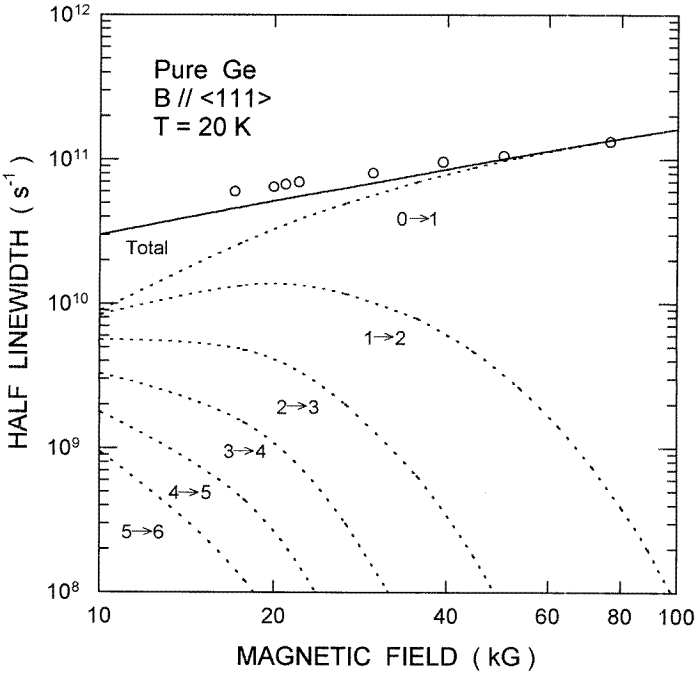

Figure 2. The magnetic field dependence of the half-CRLW of pure Ge at $T = 20$ K and with the magnetic field along the $\langle 111 \rangle$ direction. The solid line shows the total half-CRLW. The dotted lines denote the half-CRLWs due to the cyclotron transitions between the Landau levels N and $N + 1$. The open circles show the experimental data of Kobori *et al* (1990).

The magnetic field dependence of the half-CRLW for pure Ge is shown in figure 2. We see that the half-CRLW for pure Ge increases slightly with the magnetic field intensity. The maximum power intensity in equation (2.8) is related to the number density of the electrons and the half-CRLW. The maximum power intensity may be affected mainly by the number density of the electrons, since the concentration is a very fast-varying function in comparison with the half-CRLW. As the magnetic field intensity increases, the number density is decreased and the maximum power intensity decreases but the linewidth becomes broader. So, we see that the interaction between the electrons and the acoustic phonons becomes stronger as the magnetic field intensity increases, since the average energy per electron increases.

5. Concluding remarks

Using the state-independent projection operator technique (Cho and Choi 1996), we have obtained the half-CRLW formula in which the cyclotron transitions between the higher Landau levels are considered. We have applied this formula to pure Ge and obtained the temperature and the magnetic field dependence of the half-CRLW.

For temperatures below about 50 K, the present quantum-limit formula describes the

experimental data well. When the cyclotron transitions for the higher Landau levels are considered, the theoretical result is improved with only a small discrepancy. This discrepancy may stem from the scattering of the longitudinal optical deformation potential.

For metals and semiconductors with degenerate electron states, the cyclotron resonance is mainly controlled by the electron transitions between two neighbouring Landau levels near the Fermi surface. However, for semiconductors with non-degenerate electron states, the electron transitions between two lowest Landau levels provide the main contribution to the cyclotron transition. The cyclotron transitions for the higher Landau levels may be negligible for the region of the quantum limit but should be considered for the region of the classical limit as shown in figure 1. Therefore, the present calculation is more rigorous than previous ones (Bagguley *et al* 1962, Cho and Choi 1994, Ito *et al* 1964, Kobori *et al* 1990, Stradling and Zhukov 1966).

The half-CRLW increases slightly with the magnetic field intensity. Therefore, we argue that the electron–phonon interaction becomes stronger as the magnetic field intensity increases, since the average energy per electron increases.

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